17.06.2015

Quantum Field Theory II - Assignment 9

Problem 9.1 (Renormalisation of Yukawa theory)

Consider the pseudoscalar Yukawa Lagrangian

$$\mathcal{L}_{YT} = \frac{1}{2}(\partial\phi_0)^2 - \frac{1}{2}m_{0,\phi}\phi_0^2 + \bar{\Psi}_0(i\not\partial - m_{0,\psi})\Psi_0 - ig_0\phi_0\bar{\Psi}_0\gamma^5\Psi_0,$$

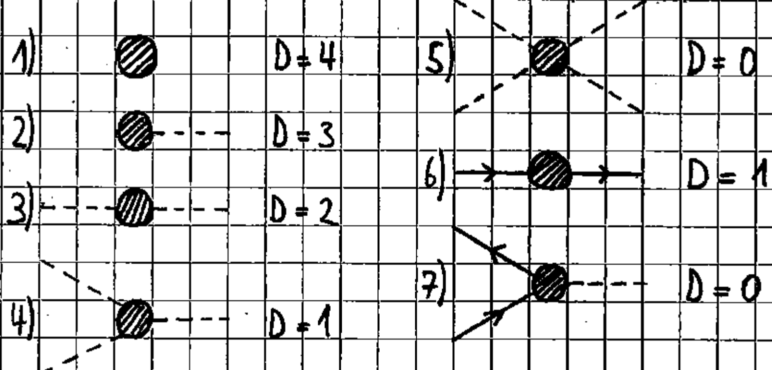
where ϕ_0 is a real scalar field and Ψ_0 is a Dirac fermion. This Lagrangian is invariant under the parity transformation $\Psi(t,\vec{x}) \longrightarrow \gamma^0\Psi(t,-\vec{x})$ and $\phi(t,\vec{x}) \longrightarrow -\phi(t,-\vec{x})$.

a) Determine the superficially divergent amplitudes and work out the Feynman rules for renormalized perturbation with this Lagrangian. Show that the theory contains a superficially divergent 4ϕ-amplitude.

Since Yukawa theory differs from QED only to the extent that the gauge field A^μ is replaced by the scalar field ϕ, Yukawa theory's superficial degree of divergence (SDD) D may be obtained from QED's by the same substitution.

$$D_{QED}^{d=4} = 4 - E_\gamma - \frac{3}{2}E_\psi \implies D_{YT}^{d=4} = 4 - E_\phi - \frac{3}{2}E_\psi =: D.$$

Since E_ϕ, E_ψ appear with a negative sign in D, we may conclude that Yukawa theory possesses only a finite number of superficially divergent amplitudes, seven to be precise. They are



All other Yukawa theory amplitudes are superficially finite.
Of these seven we may immediately discard amplitudes 2) and 4) as harmless since we know all purely scalar diagrams with an odd number of external legs to vanish due to the Lagrangians invariance under parity.
The remaining five amplitudes are remedied by absorbing their divergencies into the following counterterms,

1): absorbed into the free-to-choose vacuum energy density in a theory without gravity

3): $\delta_{Z_\phi}, \delta_{m_\phi}$ 5): δ_λ 6): $\delta_{Z_\psi}, \delta_{m_\psi}$ 7) δ_g,

which we may use to rewrite the Lagrangian as

$$\mathcal{L}_{YT} = \mathcal{L}_{YT,r} + \mathcal{L}_{YT,ct} + \delta\mathcal{L}$$

$$= \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 + \bar{\Psi}(i\not{\partial} - m_\psi)\Psi - ig\phi\bar{\Psi}\gamma^5\Psi$$

$$+ \frac{1}{2}\delta_{Z_\phi}(\partial\phi)^2 - \frac{1}{2}\delta_{m_\phi}\phi^2 - \frac{\delta_\lambda}{4!}\phi^4 + \bar{\Psi}(i\delta_{Z_\psi}\not{\partial} - \delta_{m_\psi})\Psi - i\delta_g\phi\bar{\Psi}\gamma^5\Psi,$$

in which we employed the renormalized fields

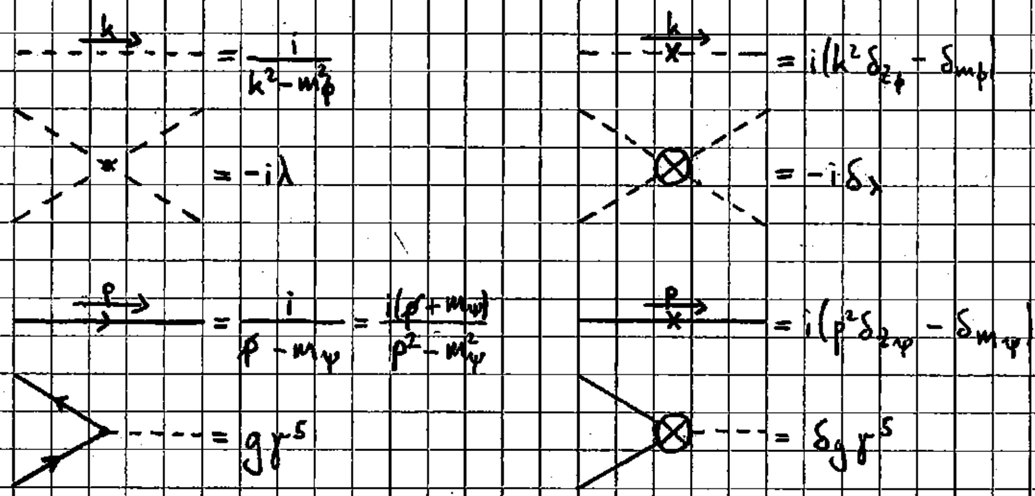
$$\phi = Z_\phi^{-\frac{1}{2}}\phi_0 \qquad \Psi = Z_\psi^{-\frac{1}{2}}\Psi_0$$

and added a ϕ^4-interaction and -counterterm, the former to remain 'natural' and the latter to deal with the divergence in amplitude 5). In this notation, the counterterms are given by

$$\delta_{Z_\phi} = Z_\phi - 1 \qquad \delta_{m_\phi} = m_0^2 Z_\phi - m^2 \qquad \delta_\lambda = \lambda_0 Z_\phi^2 - \lambda$$

$$\delta_{Z_\psi} = Z_\psi - 1 \qquad \delta_{m_\psi} = m_{0,\psi}^2 Z_\psi - m_\psi^2 \qquad \delta_g = g_0 Z_\phi^{\frac{1}{2}} Z_\psi - g$$

Thus, looking at the Lagrangian, we see that the theory requires a total of eight Feynman rules.



$= \frac{i}{k^2-m_\phi^2}$ $\qquad = i(k^2\delta_{Z_\phi} - \delta_{m_\phi})$

$= -i\lambda$ $\qquad = -i\delta_\lambda$

$= \frac{i}{\not p - m_\psi} = \frac{i(\not p + m_\psi)}{p^2 - m_\psi^2}$ $\qquad = i(\not p\,\delta_{Z_\psi} - \delta_{m_\psi})$

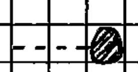
$= g\gamma^5$ $\qquad = \delta_g \gamma^5$

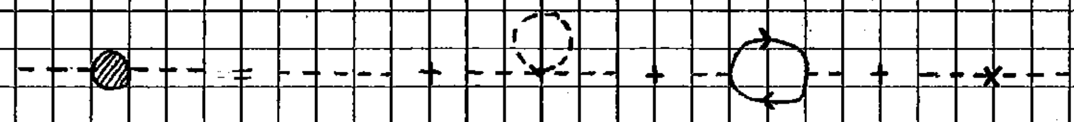
Since we considered all superficially divergent amplitudes, the above Feynman rules cover all possible diagrams in Yukawa theory. No further interactions are required.

b) Compute the divergent part (the pole as $d \to 4$) of each counterterm, to one-loop order of perturbation theory, implementing the renormalization conditions specified in part a). Disregard finite parts of the counterterms. Since the divergent parts must have a fixed dependence on the external momenta, you can simplify this calculation by choosing the momenta the simplest possible way.

In order to compute the divergent part of each of the four counterterms introduced in part a), we follow the standard renormalization procedure of calculating the divergent amplitudes to a given order in perturbation theory, in this case one-loop, and then specifying their divergencies by imposing renormalization conditions such that the previously divergent amplitudes are rendered finite.

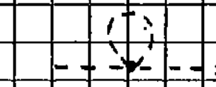
 : trivially absorbable into V_1, no renormalization required

 : vanishes due to the action's invariance under parity, also no renormalization required



+ 'higher order corrections'

The first contribution, - - - -, is finite and does not need to be considered further. Applying dimensional regularization with $d=4-\varepsilon$, we get



$$= -\frac{i}{2}\lambda\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2-m_\phi^2+i\epsilon} \overset{k^0=ik_E^0}{=} -\frac{i\lambda}{2}\int\frac{d^dk_E}{(2\pi)^d}\frac{1}{k_E^2+m_\phi^2} = -\frac{i\lambda}{2(2\pi)^d}\int_0^\infty dk_E\int d\Omega_d\,\frac{k_E^3}{k_E^2+m_\phi^2}$$

symmetry factor

Here, we can make use of

$$\sqrt{\pi}^d = \left(\int_{-\infty}^{\infty}dx\,e^{-x^2}\right)^d = \int d^dx\,e^{-\sum_{i=1}^{d}x_i^2} = \int d\Omega_d\int_0^\infty dx\,x^{d-1}e^{-x^2},\quad t=x^2,\ dt=2x\,dx$$

$$= \int d\Omega_d\int_0^\infty\frac{dt}{2}\,t^{\frac{d}{2}-1}e^{-t} = \int\Omega_d\,\frac{1}{2}\Gamma\left(\frac{d}{2}\right) \Longrightarrow \int\Omega_d = \frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}$$

$$\int_0^\infty dk_E\,\frac{k_E^{d-1}}{k_E^2+m_\phi^2} = \int_1^0\frac{(k_E^2+m_\phi^2)^2}{-2m_\phi^2k_E}\,\frac{k_E^{d-1}}{k_E^2+m_\phi^2}\,dx \qquad x=\frac{m_\phi^2}{k_E^2+m_\phi^2},\ dx=\frac{-2m_\phi^2k_E}{(k_E^2+m_\phi^2)^2}\,dk_E$$

$$= \frac{m_\phi^{d-2}}{2}\int_0^1\frac{\left(\frac{1}{x}-1\right)^{\frac{d}{2}-1}}{x}\,dx = \frac{m_\phi^{d-2}}{2}\int_0^1 x^{-\frac{d}{2}}(1-x)^{\frac{d}{2}-1}\,dx = \frac{m_\phi^{d-2}}{2}\,B\left(\alpha=-\frac{d}{2}+1,\ \beta=\frac{d}{2}\right),$$

where $B\left(\alpha=-\frac{d}{2}+1,\ \beta=\frac{d}{2}\right) = \frac{\Gamma\left(-\frac{d}{2}+1\right)\Gamma\left(\frac{d}{2}\right)}{\Gamma\left(-\frac{d}{2}+1+\frac{d}{2}\right)} = \Gamma\left(1-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right)$. Thus,

$$= -\frac{i\lambda}{2(2\pi)^d}\,\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\,\frac{m_\phi^{d-2}}{2}\,\Gamma\left(1-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}\right) = -\frac{i\lambda m_\phi^{d-2}}{2(4\pi)^{\frac{d}{2}}}\,\Gamma\left(1-\frac{d}{2}\right),$$

where $\Gamma\left(1-\frac{d}{2}\right) = \frac{\Gamma\left(2-\frac{d}{2}\right)}{1-\frac{d}{2}} = \frac{\Gamma\left(\frac{\varepsilon}{2}\right)}{\frac{\varepsilon}{2}-1} = \frac{1}{\frac{\varepsilon}{2}-1}\left(\frac{2}{\varepsilon}-\gamma+\mathcal{O}(\varepsilon)\right) = -\sum_{n=0}^{\infty}\left(\frac{\varepsilon}{2}\right)^n\left(\frac{2}{\varepsilon}-\gamma+\mathcal{O}(\varepsilon)\right)$

$$= -\left(\frac{2}{\varepsilon}-\gamma+1+\mathcal{O}(\varepsilon)\right).$$

Putting everything together, we find

$$\overset{d=4}{=} \frac{i\lambda m_\phi^2}{(4\pi)^2}\left(\frac{1}{\varepsilon}-\frac{\gamma}{2}+\frac{1}{2}\right)$$

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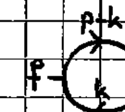
Now on to the fermionic loop,



$$= ^{*,} - g^2 \int \frac{d^dk}{(2\pi)^d} \mathrm{Tr}\left(\frac{i(\not k + m_\psi)}{k^2 - m_\psi^2 + i\epsilon} \gamma^5 \frac{i(\not k + \not p + m_\psi)}{(k+p)^2 - m_\psi^2 + i\epsilon} \gamma^5 \right)$$

$$= g^2 \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{Tr}\left((\not k + m_\psi)\gamma^5(\not k + \not p + m_\psi)\gamma^5\right)}{(k^2 - m_\psi^2 + i\epsilon)((k+p)^2 - m_\psi^2 + i\epsilon)},$$

where we use the notation Tr for a trace in spinor space and tr for a trace over space-time indices. The overall sign stems from the fermionic loop, which reads (up to commutative bosonic factors)

$$D = \bar\Psi_1 \Psi_1 \cdot \bar\Psi_2 \Psi_2 \cdot \ldots \cdot \bar\Psi_n \Psi_n = (-1)^{2n-1} \mathrm{Tr}\left(\Psi_1 \bar\Psi_2 \cdot \Psi_2 \bar\Psi_3 \cdot \ldots \cdot \Psi_n \bar\Psi_1\right),$$

for a loop with n vertices along its arc. We commuted $\bar\Psi_1$ through to the end earning an odd number of $2n-1$ signs to obtain n propagators and a trace since the propagators meet meet up after running around the loop, closing the summation over Dirac matrix indices. (In the path integral approach, the Ψ_i are anticommuting Grassmann numbers.)

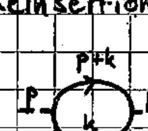
We first simplify the numerator using gamma-matrix trace identities,

$$\mathrm{Tr}\left((\not k + m_\psi)\gamma^5(\not k + \not p + m_\psi)\gamma^5\right) = \mathrm{Tr}\left(\not k \gamma^5 (\not k + \not p)\gamma^5\right) + \mathrm{Tr}\left(\not k \gamma^5 m_\psi \gamma^5\right)$$

$$+ \mathrm{Tr}\left(m_\psi \gamma^5 (\not k + \not p)\gamma^5\right) + \mathrm{Tr}\left(m_\psi^2 \underbrace{(\gamma^5)^2}_{\mathbb{1}_4}\right) = -\mathrm{Tr}\left(\not k^2 + \not k \not p\right) + m_\psi \underbrace{\mathrm{Tr}(\not k)}_{0}$$

$$+ m_\psi \underbrace{\mathrm{Tr}(\not k + \not p)}_{0} + 4 m_\psi^2 = -4\left(k^2 + kp - m_\psi^2\right).$$

Reinsertion gives



$$= -4g^2 \int \frac{d^dk}{(2\pi)^d} \frac{k^2 + kp - m_\psi^2}{(k^2 - m_\psi^2 + i\epsilon)((k+p)^2 - m_\psi^2 + i\epsilon)}$$

$$= -4g^2 \int \frac{d^dk}{(2\pi)^d} \int_0^1 dx \frac{k^2 + kp - m_\psi^2}{\left[x((k+p)^2 - m_\psi^2 + i\epsilon) + (1-x)(k^2 - m_\psi^2 + i\epsilon)\right]^2}$$

*, Remark: Diagrams of Yukawa theory (and QED) always have symmetry factor one, since the fields $\phi\bar\Psi\Psi$ ($A_\mu \bar\Psi\Psi$) cannot substitute for one another in contractions.

where we used the identity (involving the so-called Feynman parameter x),

$$\frac{1}{AB}=\int_0^1 dx\,\frac{1}{(xA+(1-x)B)^2},$$

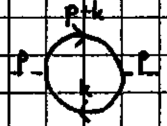
with $A=(k+p)^2-m_\psi^2+i\epsilon$ and $B=k^2-m_\psi^2+i\epsilon$ in our case. Having thus combined the denominator, we complete the square in k and substitute,

$$x((p+k)^2-m_\psi^2+i\epsilon)+(1-x)(k^2-m_\psi^2+i\epsilon)=x(p^2+2pk)+x(k^2-m_\psi^2+i\epsilon)$$

$$+(k^2-m_\psi^2+i\epsilon)-x(k^2-m_\psi^2+i\epsilon)=k^2+2xpk+xp^2-m_\psi^2+i\epsilon+x^2p^2-x^2p^2$$

$$=(k+xp)^2+x(1-x)p^2-m_\psi^2+i\epsilon = l^2+\Delta,$$

to rewrite the integral as

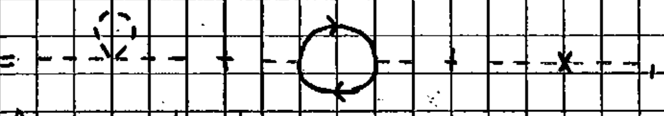


$$= -4g^2\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\,\frac{k^2+kp-m_\psi^2}{(l^2+\Delta)^2}\overset{\text{How?!}}{=}-4g^2\int_0^1 dx\int\frac{d^dl}{(2\pi)^d}\,\frac{l^2-x(1-x)p^2+m_\psi^2}{(l^2+\Delta)^2}$$

$$=-4g^2\int_0^1 dx\,\frac{-i}{(4\pi)^{\frac d2}}\left(\frac{\frac d2\Gamma(1-\frac d2)}{(-\Delta)^{1-d/2}}+\frac{\Delta\Gamma(2-\frac d2)}{(-\Delta)^{2-\frac d2}}\right)=\frac{4ig^2(d-1)}{(4\pi)^{d/2}}\int_0^1 dx\,\frac{\Gamma(1-\frac d2)}{(-\Delta)^{1-\frac d2}}$$

$$=\frac{4ig^2(p^2-2m_\psi^2)}{(4\pi)^2}\frac{1}{\epsilon}$$

Since $-iM^2(p^2)=$ (1PI) $=$



our usual choice of renormalization conditions

$$\text{i) } M^2(p^2)\big|_{p^2=m_\phi^2}=0 \quad\text{and}\quad \text{ii) } \frac{d}{dp^2}M^2(p^2)\Big|_{p^2=m_\phi^2}=0,$$

amounts to the following counterterms

$$-iM^2(p^2=m_\phi^2)=\frac{i\lambda m_\phi^2}{(4\pi)^2}\left(\frac1\epsilon-\frac\gamma2+\frac12\right)+\frac{4ig^2}{(4\pi)^2}(p^2-2m_\psi^2)\frac1\epsilon+i(p^2\delta_{Z_\phi}-\delta m_\phi)\Big|_{p^2=m_\phi^2}$$

$$=\frac{i\lambda m_\phi^2}{(4\pi)^2}\frac1\epsilon+\frac{4ig^2}{(4\pi)^2}(m_\phi^2-2m_\psi^2)\frac1\epsilon+i(m_\phi^2\delta_{Z_\phi}-\delta m_\phi)+\text{"finite terms"}\overset{!}{=}0$$

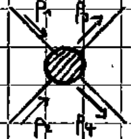
$$\frac{d}{dp^2}(-iM^2(p^2))\Big|_{p^2=m_\phi^2}=\frac{4ig^2}{(4\pi)^2}\frac1\epsilon+i\delta_{Z_\phi}\overset{!}{=}0\implies \delta_{Z_\phi}=-\frac{g^2}{4\pi^2}\frac1\epsilon,\ \delta_{m_\phi}=\frac{(\lambda m_\phi^2-8g^2m_\psi^2)}{(4\pi)^2}\frac1\epsilon$$

Problem 9.2 (Asymptotic behavior of diagrams in ϕ^4-theory)

Compute the leading terms in the S-matrix element for boson-boson scattering in ϕ^4-theory in the limit $s \to \infty$, t fixed. Ignore all masses on internal lines and keep external masses non-zero only as infrared regulators where these are needed. Show that

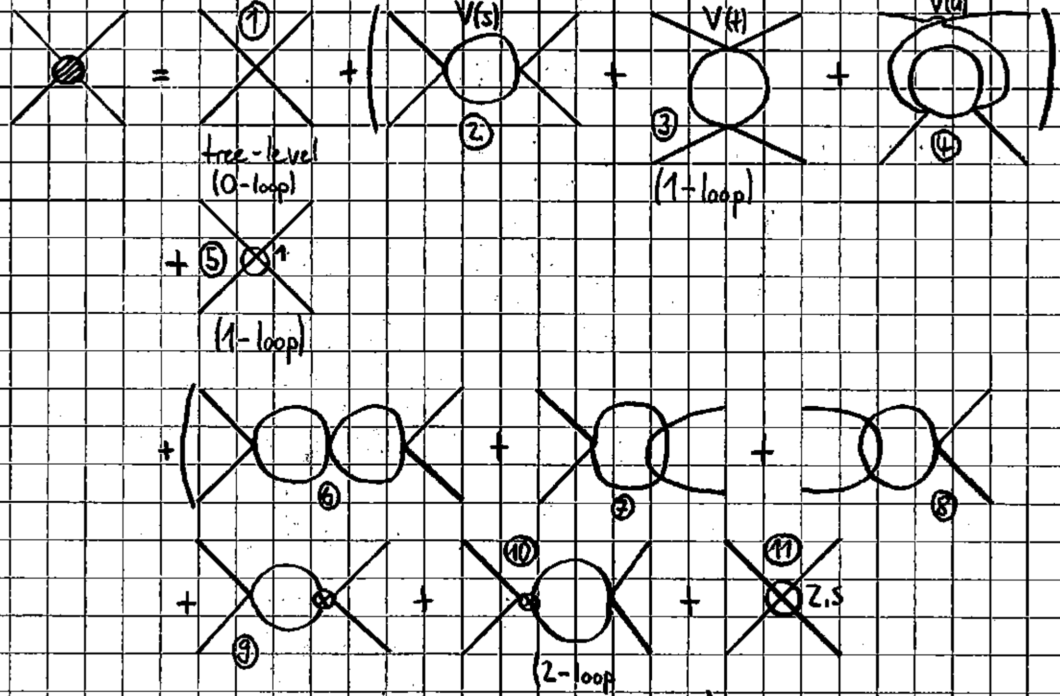
$$i\mathcal{M}(s,t) \propto -i\lambda - \frac{i\lambda^2}{(4\pi)^2}\log(s) - \frac{5i\lambda^3}{2(4\pi)^4}\log^2(s) + \dots \qquad (3)$$

The process of boson-boson scattering in $\lambda\phi^4$-theory is represented by



$= i\mathcal{M}(p_1, p_2 \to p_3, p_4).$

We will investigate all elements of a perturbative expansion of this process up to the two-loop level. The expansion yields



$+ \text{(t-channel)} + \text{(u-channel)}\Big)$

Before proceeding, we note the implications of the limit $s \to \infty$, t fixed.

$$s+t+u = 2m^2 + 2p_1p_2 + 2m^2 - 2p_1p_3 + 2m^2 - 2p_1p_4 = 6m^2 + 2p_1(\underbrace{p_2 - p_3 - p_4}_{-p_1})$$
$$= 4m^2.$$

Thus as $s \to \infty$, t and $4m^2$ both remain constant quickly becoming irrelevant, so that $s \approx -u$.

Returning to our loop expansion, we make use of the fact that the first five diagrams (tree- and 1-loop level) were already computed as part of problem 7.1 b). We briefly recall our findings:

$$i\mathcal{M}(p_1,p_2 \to p_3,p_4)\Big|_{1\text{-loop}} = -i\lambda + \frac{(i\lambda)^2}{2}\int \frac{d^dk}{(2\pi)^d} \frac{i}{k^2-m^2}\left[\frac{i}{(p_s-k)^2-m^2} + \frac{i}{(p_t-k)^2-m^2} + \frac{i}{(p_u-k)^2-m^2}\right] - i\delta_\lambda$$

$$\approx \frac{i\lambda^2}{2(4\pi)^2}\left[3\left(\frac{2}{\epsilon}-\gamma+\log(4\pi)\right)-\log(s)-\log(t)-\log(u)\right]-i\delta_\lambda - i\lambda$$

$$= -i\lambda - \frac{i\lambda^2}{2(4\pi)^2}\left[\log(s)+\log(t)+\underbrace{\log(u)}_{\log(-s)=\log(s)+i\pi}\right] \approx -\frac{i\lambda^2}{(4\pi)^2}\log(s), \quad \text{for } s\to\infty,\ t \text{ fixed.}$$

We set up the δ_λ coefficient at 1-loop order to absorb the divergence

$$\delta_\lambda = \frac{3\lambda^2}{2(4\pi)^2}\left(\frac{2}{\epsilon}-\gamma+\log(4\pi)\right).$$

It contains finite terms which we could have left as part of $i\mathcal{M}(p_1,p_2\to p_3,p_4)$. Absorbing additional constants besides the actual (momentum-independent) divergencies into counterterms merely results in a change to the theory's parameter this particular counterterm was set up to renormalize (in this instance λ).

Note: We will also encounter momentum-dependent divergencies. These always need to cancel among all diagrams of a given loop order.

We now tread on untarnished ground by calculating all two-loop corrections.

$$⑥ = \frac{(-i\lambda)^3}{4}\left[\int \frac{d^dk}{(2\pi)^d}\frac{i}{k^2}\frac{i}{(p_s-k)^2}\right]^2 = (-i\lambda)^3 V^2(p^2) = (-i\lambda)^3\left(\frac{-1}{2(4\pi)^2}\left(\frac{2}{\epsilon}-\gamma+\ln(4\pi)-\ln(s)\right)\right)^2$$

$$\approx (-i\lambda)^3\left[\frac{-1}{(4\pi)^2}\left(\frac{1}{\epsilon}-\frac{1}{2}\ln(s)\right)\right]^2 = -\frac{i\lambda}{(4\pi)^4}\left(\frac{1}{\epsilon^2}-\frac{1}{\epsilon}\ln(s)+\frac{1}{4}\ln^2(s)\right)$$

$$⑦ = ⑧ = \frac{(-i\lambda)^3}{2}\int\frac{d^dk}{(2\pi)^d}\int\frac{d^dq}{(2\pi)^d}\frac{i}{k^2}\frac{i}{(p_s+k)^2}\frac{i}{q^2}\frac{i}{(k+p_3+q)^2}$$

$$= (-i\lambda)^3\int\frac{d^dk}{(2\pi)^d}\frac{i}{k^2}\frac{i}{(k+p_s)^2}\, i V((k+p_3)^2) = \frac{\lambda^3(-1)^{2-\frac{d}{2}}\Gamma(2-\frac{d}{2})}{2(4\pi)^{d/2}}\underbrace{\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2}\frac{1}{(k+p_s)^2}\frac{1}{((k+p_3)^2)^{2-\frac{d}{2}}}}_{\Sigma}$$

$$\Sigma = \int_0^1 dy\int_0^1 dz\,\frac{z^{1-\frac{d}{2}}}{(l^2-\Delta)^{4-\frac{d}{2}}}\underbrace{\frac{\Gamma(4-\frac{d}{2})}{\Gamma(2-\frac{d}{2})}}_{\beta},\quad \text{where } l = k+yp_s+zp_3$$

in the limit $s \to \infty$

$$\Delta = 2yz\,p_sp_3 - y(1-y)p_s^2 \approx y(y+z-1)s$$

$$= \int_0^1\frac{dy}{y^\epsilon}\int_0^1\frac{dz}{z^{1-\frac{\epsilon}{2}}}\frac{1}{(y-z-1)^2}\,\beta$$

$$= \beta\int_0^1 dy\,y^{-\epsilon}(y-1)^{-\epsilon}\int_0^1\frac{dz}{z^{1-\frac{\epsilon}{2}}} = \beta\,\frac{\Gamma(1-\epsilon)\Gamma(1-\epsilon)}{\Gamma(1-\epsilon+1-\epsilon)}\overbrace{(-1)^{-\epsilon}}^{\xrightarrow{\epsilon\to 0} 1}\left(1-\left(1-\frac{\epsilon}{2}\right)\right)^{-1}$$

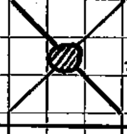
$$⑦ = ⑧ = -\frac{i\lambda^3}{(4\pi)^4}\Gamma(\epsilon)\frac{1}{s^\epsilon}\frac{1}{\epsilon} = -\frac{i\lambda}{(4\pi)^4}\left(\frac{1}{\epsilon}-\gamma+\sigma(\epsilon)\right)\left(1-\epsilon\ln(s)+\frac{1}{2}\epsilon^2\ln^2(s)+\sigma(\epsilon^3)\right)\frac{1}{\epsilon}$$

$$= -\frac{i\lambda^3}{(4\pi)^4}\left(\frac{1}{\epsilon^2}-\frac{1}{\epsilon}\ln(s)+\frac{1}{2}\ln^2(s)-\underbrace{\cancel{\frac{\gamma}{\epsilon}}}_{\ll \ln(s)}+\sigma(\epsilon)\right)$$

$$⑨ = ⑩ = (-i\lambda)(-i\delta_\lambda^1)\,iV(p^2),\quad \text{where}\quad V(p^2) = -\frac{\Gamma(\epsilon/2)}{2(4\pi)^2}\frac{1}{(p^2)^{\epsilon/2}}\quad\text{and}\quad \delta_\lambda^1 = \frac{3\lambda}{(4\pi)^2}\frac{1}{\epsilon}$$

$$= \frac{3\lambda^2}{(4\pi)^2}\left(\frac{1}{\epsilon^2}-\frac{1}{2\epsilon}+\frac{1}{8}\ln^2(s)\right)$$

$$\sum_{\substack{\text{sum over 2-loop}\\ \text{diagrams}}}(s\text{-channel}) = \frac{i\lambda^3}{(4\pi)^4}\left(\frac{3}{\epsilon^2}-\frac{3}{4}\ln^2(s)\right)$$



$$= -i\lambda - \frac{i\lambda^2}{(4\pi)^2}\ln(s) - \frac{3i\lambda^2}{2(4\pi)^2}\ln^2(s)$$