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General Relativity - Exercise Sheet 8

Problem 1 (Spherical symmetry and time derivatives) [15 points]

a) Argue why the free Einstein field equations can be written as

$$R_{\mu\nu} = 0, \tag{1}$$

where $R_{\mu\nu}$ are the entries of the Ricci-tensor.

For a vacuum with vanishing energy density, i.e. cosmological constant $\Lambda = 0$, the Einstein field equations become

$$R_{\mu\nu} = -\frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}\right).$$

In the free case, i.e. a vacuum devoid of any energy-matter distribution, we set the energy-momentum tensor $T_{\mu\nu}$ (and consequently its trace T) to zero, rendering the field eqs.

$$R_{\mu\nu} = 0.$$

b) Given the metric

$$\operatorname{diag}(g) = \left(B(r,t), -A(r,t), -r^2, -r^2\sin^2\theta\right),$$

compute the entries R_{00}, R_{11} and R_{10} of the Ricci-tensor

$$R_{\mu\nu} := \Gamma^\rho_{\mu\rho,\nu} - \Gamma^\rho_{\mu\nu,\rho} + \Gamma^\sigma_{\mu\rho}\Gamma^\rho_{\sigma\nu} - \Gamma^\sigma_{\mu\nu}\Gamma^\rho_{\sigma\rho}.$$

$$R_{00} = \partial_0 \Gamma^\rho_{0\rho} - \partial_\rho \Gamma^\rho_{00} + \Gamma^\sigma_{0\rho}\Gamma^\rho_{\sigma 0} - \Gamma^\sigma_{00}\Gamma^\rho_{\sigma\rho}$$

$$= \cancel{\partial_0 \Gamma^0_{00}} + \partial_0 \Gamma^1_{01} - \cancel{\partial_0 \Gamma^0_{00}} - \partial_1 \Gamma^1_{00} + \cancel{\Gamma^0_{00}\Gamma^0_{00}} + \Gamma^0_{01}\Gamma^1_{00}$$

$$+ \cancel{\Gamma^1_{00}\Gamma^0_{10}} + \Gamma^1_{01}\Gamma^1_{10} - \cancel{\Gamma^0_{00}\Gamma^0_{00}} - \Gamma^0_{00}\Gamma^1_{01} - \cancel{\Gamma^1_{00}\Gamma^0_{10}}$$

$$- \Gamma^1_{00}\Gamma^1_{11} - \Gamma^1_{00}\Gamma^2_{12} - \Gamma^1_{00}\Gamma^3_{13}$$

$$= \partial_t \frac{\dot{A}}{2A} - \partial_r \frac{B'}{2A} + \frac{B'}{2B}\frac{B'}{2A} + \frac{\dot{A}}{2A}\frac{\dot{A}}{2A} - \frac{\dot{B}}{2B}\frac{\dot{A}}{2A} - \frac{B'}{2A}\frac{A'}{2A} - \frac{B'}{2A}\frac{1}{r} - \frac{B'}{2A}\frac{1}{r}$$

$$= \frac{\ddot{A}}{2A} - \frac{\dot{A}^2}{2A^2} - \frac{B''}{2A} + \frac{A'B'}{2A^2} + \frac{B'^2}{4AB} + \frac{\dot{A}^2}{4A^2} - \frac{\dot{A}\dot{B}}{4AB} - \frac{A'B'}{4A^2} - \frac{B'}{Ar}$$

$$= \frac{\ddot{A} - B''}{2A} + \frac{A'B' - \dot{A}^2}{4A^2} + \frac{B'^2 - \dot{A}\dot{B}}{4AB} - \frac{B'}{Ar}$$

$$R_{11} = \partial_1 \Gamma^{\rho}_{1\rho} - \partial_\rho \Gamma^{\rho}_{11} + \Gamma^{\sigma}_{1\rho}\Gamma^{\rho}_{\sigma 1} - \Gamma^{\sigma}_{11}\Gamma^{\rho}_{\sigma\rho}$$

$$= \partial_1 \Gamma^0_{10} + \cancel{\partial_1 \Gamma^1_{11}} + \partial_1 \Gamma^2_{12} + \partial_1 \Gamma^3_{13} - \partial_0 \Gamma^0_{11} - \cancel{\partial_1 \Gamma^1_{11}}$$

$$+ \Gamma^0_{10}\Gamma^0_{01} + \cancel{\Gamma^0_{11}\Gamma^1_{01}} + \Gamma^1_{10}\Gamma^0_{11} + \cancel{\Gamma^1_{11}\Gamma^1_{11}} + \Gamma^2_{12}\Gamma^2_{21}$$

$$+ \Gamma^3_{13}\Gamma^3_{31} - \Gamma^0_{11}\Gamma^0_{00} - \cancel{\Gamma^0_{11}\Gamma^1_{01}} - \Gamma^1_{11}\Gamma^0_{10} - \cancel{\Gamma^1_{11}\Gamma^1_{11}}$$

$$- \Gamma^1_{11}\Gamma^2_{12} - \Gamma^1_{11}\Gamma^3_{13}$$

$$= \partial_r \frac{B'}{2B} + \partial_r \frac{1}{r} + \partial_r \frac{1}{r} - \partial_t \frac{\dot{A}}{2B} + \frac{B'}{2B}\frac{B'}{2B} + \frac{\dot{A}}{2A}\frac{\dot{A}}{2B} + \frac{1}{r}\frac{1}{r} + \frac{1}{r}\frac{1}{r}$$

$$- \frac{\dot{A}}{2B}\frac{\dot{B}}{2B} - \frac{A'}{2A}\frac{B'}{2B} - \frac{A'}{2A}\frac{1}{r} - \frac{A'}{2A}\frac{1}{r}$$

$$= \frac{B''}{2B} - \frac{B'^2}{2B^2} - \cancel{\frac{2}{r^2}} - \frac{\ddot{A}}{2B} + \frac{\dot{A}\dot{B}}{2B^2} + \frac{B'^2}{4B^2} + \frac{\dot{A}^2}{4AB} + \cancel{\frac{2}{r^2}} - \frac{\dot{A}\dot{B}}{4B^2} - \frac{A'B'}{4AB} - \frac{A'}{Ar}$$

$$= \frac{B'' - \ddot{A}}{2B} + \frac{\dot{A}\dot{B} - B'^2}{4B^2} + \frac{\dot{A}^2 - A'B'}{4AB} - \frac{A'}{Ar}$$

$$R_{10} = \partial_0 \Gamma^{\rho}_{1\rho} - \partial_\rho \Gamma^{\rho}_{10} + \Gamma^{\sigma}_{1\rho}\Gamma^{\rho}_{\sigma 0} - \Gamma^{\sigma}_{10}\Gamma^{\rho}_{\sigma\rho}$$

$$= \cancel{\partial_0 \Gamma^0_{10}} + \partial_0 \Gamma^1_{11} + \partial_0 \Gamma^2_{12} + \partial_0 \Gamma^3_{13} - \cancel{\partial_0 \Gamma^0_{10}} - \partial_1 \Gamma^1_{10}$$

$$+ \cancel{\Gamma^0_{10}\Gamma^0_{00}} + \Gamma^0_{11}\Gamma^1_{00} + \cancel{\Gamma^1_{10}\Gamma^0_{10}} + \Gamma^1_{11}\Gamma^1_{10}$$

$$- \cancel{\Gamma^0_{10}\Gamma^0_{00}} - \Gamma^0_{10}\Gamma^1_{01} - \cancel{\Gamma^1_{10}\Gamma^0_{10}} - \Gamma^1_{10}\Gamma^1_{11} - \Gamma^1_{10}\Gamma^2_{12}$$

$$- \Gamma^1_{10}\Gamma^3_{13}$$

$$= \cancel{\partial_t \frac{A'}{2A}} + \partial_t \frac{1}{r} + \partial_t \frac{1}{r} - \cancel{\partial_r \frac{\dot{A}}{2A}} + \cancel{\frac{\dot{A}}{2B}\frac{B'}{2A}} + \cancel{\frac{A'}{2A}\frac{\dot{A}}{2A}} - \cancel{\frac{B'}{2B}\frac{\dot{A}}{2A}} - \cancel{\frac{\dot{A}}{2A}\frac{A'}{2A}}$$

$$-\frac{\dot{A}}{2A}\frac{1}{r}-\frac{\dot{A}}{2A}\frac{1}{r}=-\frac{\dot{A}}{Ar}$$

c) What sort of spatial symmetry is the above metric assuming?

Symmetries of a metric are characterized by the existence of Killing vectors. Since the above metric has three Killing vectors $(V^{(1)}, V^{(2)}, V^{(3)})$ fulfilling the same commutation relations

$$[V^{(i)}, V^{(j)}] = V^{(k)}, \qquad i,j,k \in \{1,2,3\},\ i \neq j \neq k \neq i$$

as those of the two-sphere S^2, we may conclude that our metric assumes spherical symmetry.

d) For each individual entry of the Ricci-tensor eq. (1) holds. What does that say about $\dot{A}$ if you consider R_{10} in this vacuum?

Assuming $A(r,t) \neq 0 \ \forall r,t$, we infer

$$0 \stackrel{!}{=} R_{10} = -\frac{\dot{A}}{Ar} \implies \dot{A}=0,\ A(r,t)=A(r) \text{ outside the origin, i.e. } r\neq 0$$

e) Calculate $\frac{R_{00}}{B}+\frac{R_{11}}{A}$.

$$\frac{R_{00}}{B}+\frac{R_{11}}{A}=\frac{\ddot{A}-B''}{2AB}+\frac{A'B'-\dot{A}^2}{4A^2B}+\frac{B'^2-\dot{A}\dot{B}}{4AB^2}-\frac{B'}{ABr}$$

$$+\frac{B''-\ddot{A}}{2AB}+\frac{\dot{A}\dot{B}-B'^2}{4AB^2}+\frac{\dot{A}^2-A'B'}{4A^2B}-\frac{A'}{A^2r}=-\frac{B'}{ABr}-\frac{A'}{A^2r}$$

f) Using $\frac{R_{00}}{B}+\frac{R_{11}}{A}=0$ and your previous result, show that $\frac{d}{dr}\log(AB)=0$, therefore $AB=\text{const}$ (w.r.t. to change in r).

$$0 \stackrel{!}{=} \frac{R_{00}}{B}+\frac{R_{11}}{A}=-\frac{B'}{ABr}-\frac{A'}{A^2r} \stackrel{\cdot(-Ar)}{\iff} 0=\frac{A'B+AB'}{AB}=\frac{d}{dr}\log(AB)$$

Contrary to what the exercise suggests, this does not imply $\dot{B}=0$, since we only found $\dot{A}=0$, but not $A'=0$, as would be required to conclude $B'=0$!

Problem 2 (Piep piep kleiner Sattelit) [15 points]

A sattelite with mass $m>0$ is orbiting a black hole. The metric is

$$\mathrm{diag}(g) = (B(r), -A(r), -r^2, -r^2\sin^2\theta).$$

a) Calculate $\frac{d^2x^0}{d^2\lambda} = \frac{d^2t}{d^2\lambda}$ via the geodesic equation.

Since $A(r)$ and $B(r)$ are independent of t now, we may use the dot notation to indicate differentiation w.r.t. λ, i.e. $\dot{x}^\mu = \frac{dx^\mu}{d\lambda}$.

The geodesic equation then reads

$$\ddot{x}^\mu + \Gamma^\mu{}_{\rho\sigma}\,\dot{x}^\rho\dot{x}^\sigma = 0.$$

Therefore, ignoring Christoffel symbols containing $\dot{A}$ and $\dot{B}$, we get

$$\frac{d^2t}{d^2\lambda} = \ddot{x}^0 = -\Gamma^0{}_{\rho\sigma}\,\dot{x}^\rho\dot{x}^\sigma = -\Gamma^0{}_{01}\,\dot{x}^0\dot{x}^1 - \Gamma^0{}_{10}\,\dot{x}^1\dot{x}^0 = -2\Gamma^0{}_{01}\,\dot{x}^0\dot{x}^1$$

$$= -2\frac{B'}{2B}\,\dot{t}\,\dot{r} = -\frac{B'}{B}\,\dot{t}\,\dot{r}$$

✓

b) Rewrite the result as

$$\frac{d}{d\lambda}\left(\log f + \log g\right) = 0$$

thus showing that $fg =: F = \text{const}$. What are f and g?

$$B\frac{d^2t}{d^2\lambda} + \underbrace{\frac{dB}{dr}\frac{dr}{d\lambda}}_{dB/d\lambda}\frac{dt}{d\lambda} = B\frac{d^2t}{d^2\lambda} + \frac{dB}{d\lambda}\frac{dt}{d\lambda} = \frac{d}{d\lambda}\left(B\frac{dt}{d\lambda}\right) = 0$$

Thus, we found $B\frac{dt}{d\lambda} = B\dot{t}$ to be constant w.r.t. λ.

✓

c) Consider the equatorial plane, $\theta = \frac{\pi}{2}$. Show that the geodesic equation for ϕ delivers

$$r^2 \frac{d\phi}{d\lambda} \equiv L \,, \qquad L = \text{const}.$$

$$\ddot{\phi} = \ddot{x}^3 = -\Gamma^3_{\rho\sigma} \dot{x}^\rho \dot{x}^\sigma = -2\Gamma^3_{13} \dot{x}^1 \dot{x}^3 - 2\Gamma^3_{23} \dot{x}^2 \dot{x}^3$$

$$= -\frac{2}{r} \dot{r} \dot{\phi} - \underbrace{2 \cot\theta \, \dot{\theta} \dot{\phi}}_{0 \text{ for } \theta = \frac{\pi}{2}}$$

$$\Longrightarrow r^2 \frac{d^2\phi}{d^2\lambda} + \underbrace{2r \frac{dr}{d\lambda}}_{dr^2/d\lambda} \frac{d\phi}{d\lambda} = \frac{d}{d\lambda}\left(r^2 \frac{d\phi}{d\lambda} \right) = 0$$

The geodesic eq. for ϕ yields $r^2 \frac{d\phi}{d\lambda} \equiv L$ to be a constant.

d) Show that the last interesting geodesic equation, the one for r, delivers

$$\frac{d^2 r}{d^2\lambda} + \frac{F^2 B'}{2AB^2} + \frac{A'}{2A} \left(\frac{dr}{d\lambda} \right)^2 - \frac{L^2}{Ar^3} = 0$$

$$\ddot{x}^1 + \Gamma^1_{\rho\sigma} \dot{x}^\rho \dot{x}^\sigma = \ddot{x}^1 + \Gamma^1_{00} \dot{x}^0 \dot{x}^0 + \Gamma^1_{11} \dot{x}^1 \dot{x}^1 + \Gamma^1_{22} \dot{x}^2 \dot{x}^2$$

$$= \ddot{r} + \frac{B'}{2A} \dot{t}^2 + \frac{A'}{2A} \dot{r}^2 - \frac{r}{A} \dot{\theta}^2$$

$$= \frac{d^2 r}{d^2\lambda} + \frac{B'}{2AB^2} F^2 + \frac{A'}{2A} \left(\frac{dr}{d\lambda} \right)^2 - \frac{L^2}{Ar^3} = 0$$

e) Multiplying the above result by $2A \frac{dr}{d\lambda}$, show that

$$A \left(\frac{dr}{d\lambda} \right)^2 + \frac{L^2}{r^2} - \frac{F^2}{B^2} = -\epsilon = \text{const}.$$

$$2A \frac{dr}{d\lambda} \frac{d^2 r}{d^2\lambda} + B' \frac{F^2}{B^2} \frac{dr}{d\lambda} + A' \left(\frac{dr}{d\lambda} \right)^3 - \frac{2L^2}{r^3} \frac{dr}{d\lambda}$$

$$= 2A \frac{dr}{d\lambda} \frac{d^2 r}{d^2\lambda} - \frac{d}{d\lambda} \frac{F^2}{B} + \frac{dA}{d\lambda} \left(\frac{dr}{d\lambda} \right)^2 + \frac{d}{d\lambda} \frac{L^2}{r^2}$$

$$= \frac{d}{d\lambda} \left(A \left(\frac{dr}{d\lambda} \right)^2 \right) - \frac{d}{d\lambda} \frac{F^2}{B} + \frac{d}{d\lambda} \frac{L^2}{r^2} = \frac{d}{d\lambda} \Big(\underbrace{A \left(\frac{dr}{d\lambda} \right)^2 - \frac{F^2}{B} + \frac{L^2}{r^2}}_{\equiv -\epsilon = \text{const w.r.t. } \lambda} \Big) = 0$$

f) Using $B(r) = A^{-1}(r) = 1 - \frac{2a}{r}$, simplify the last result to

$$\frac{\dot{r}^2}{2} - \frac{a\epsilon}{r} + \frac{L^2}{2r^2} - \frac{aL^2}{r^3} = \frac{F^2-\epsilon}{2}.$$

We multiply our last result with $\frac{B}{2}$ and add $\frac{F^2}{2}$ to get

$$\frac{\dot{r}^2}{2} + B\frac{L^2}{2r^2} = \frac{\dot{r}^2}{2} + \frac{L^2}{2r^2} - \frac{aL^2}{r^3} = \frac{F^2}{2} - B\frac{\epsilon}{2} = \frac{F^2-\epsilon}{2} + \frac{a\epsilon}{r}$$

$$\Longrightarrow \frac{\dot{r}^2}{2} - \frac{a\epsilon}{r} + \frac{L^2}{2r^2} - \frac{aL^2}{r^3} = \frac{F^2-\epsilon}{2}$$

g) Since our satellite has mass, we assume $\lambda = \tau$ and $\epsilon = c^2$ (for photons we would have $\epsilon = 0$). For simplicity, we impose a spherical orbit. From the lecture, we know $a = GM/c^2$, and define the effective potential

$$V_{eff}(r) = -\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{c^2r^3}.$$

Which term is new when compared to Newtonian gravity?
The first term of $V_{eff}(r)$ corresponds exactly to the Newtonian gravitational potential $V_G = -\frac{GM}{r}$. The second term is a contribution from the satellite's angular momentum, also known from Newtonian gravity. The third term, however, is unfamiliar, meaning it is a proper contribution from general relativity. Obviously, it becomes important at small radial distances r.

h) What does eq. (3) look like for our sattelite?

$$\frac{dt}{d\tau} = \frac{F}{B} = \frac{1}{1-\frac{2GM}{c^2r}}\left(\dot{r}^2 - \frac{2a\epsilon}{r} + \frac{L^2}{r^2} - \frac{2aL^2}{r^3} + \epsilon\right)^{\frac{1}{2}}$$

$$\stackrel{\wedge}{=} \left(1-\frac{2GM}{c^2r}\right)^{-1}\left(c^2\left(1-\frac{2GM}{c^2r}\right) + \frac{L^2}{r^2}\left(1-\frac{2GM}{c^2r}\right)\right)^{\frac{1}{2}} = \sqrt{\frac{c^2+\dot{\varphi}^2r^2}{1-2a/r}}!$$

setting $\epsilon = c^2$ and $\dot{r} = 0$ for circular orbit

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i) Now we want to get rid of the L-dependency. In order to do this, we impose $dV_{eff}(r)/dr = 0$. Solve this for $L^2/(c^2r^2)$ to arrive at an expression for $dt/d\tau$ free of L.

$$\frac{dV_{eff}}{dr} = \frac{GM}{r^2} - \frac{L^2}{r^3} + \frac{3GML^2}{c^2r^4} = \frac{GM}{r^2} + \left(\frac{3GM}{r^2} - \frac{c^2}{r}\right)\frac{L^2}{c^2r^2} \overset{!}{=} 0$$

$$\Longrightarrow \frac{L^2}{c^2r^2} = \frac{-GM}{3GM - c^2r} = \frac{1}{\frac{c^2r}{GM} - 3}$$

Inserting this relation into our result from part h), we obtain

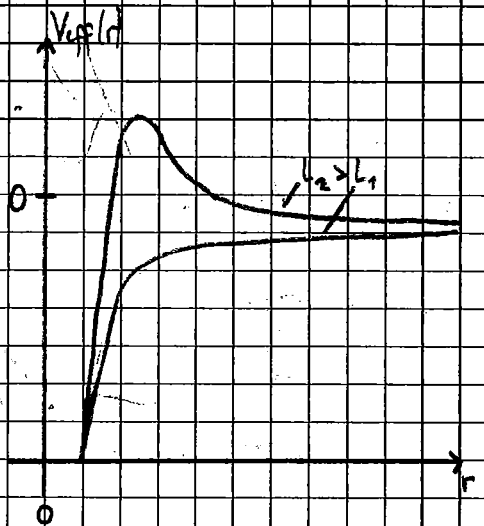
$$\frac{dt}{d\tau} = \left(1 - \frac{2GM}{c^2r}\right)^{-1}\left(c^2\left(1 - \frac{2GM}{c^2r}\right)\left(1 + \frac{L^2}{c^2r^2}\right)\right)^{\frac{1}{2}}$$

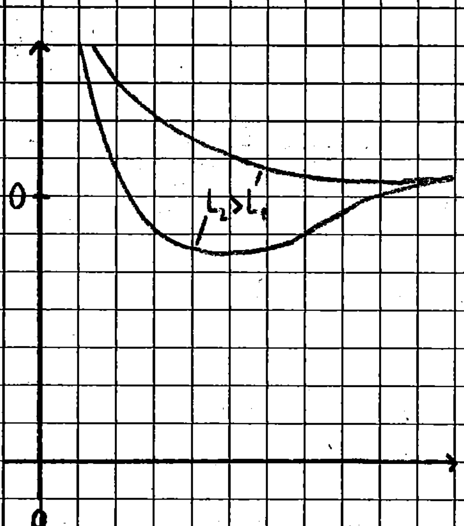
$$= \left(1 - \frac{2GM}{c^2r}\right)^{-1}\left(c^2\left(1 - \frac{2GM}{c^2r}\right)\left(1 - \frac{GM}{3GM - c^2r}\right)\right)^{\frac{1}{2}} \Longrightarrow$$

$$d\tau = \left(1 - \frac{3GM}{rc^2}\right)^{1/2} dt$$

j) Extra: Plot $V_{eff}(r)$ and $\tilde{V}_{eff}(r)$. Is there anything surprising?

Plots of $V_{eff}(r)$ and $\tilde{V}_{eff}(r)$ for different L.





These plots are very different from those drawn for Newtonian gravity. For one, the potential goes to $-\infty$, rather than $+\infty$, as $r \to 0$. Secondly, $V_{eff}(r)$ exhibits maxima (whereas $V_{Newt}(r)$ may only have minima or no extrema at all, depending on L), allowing for unstable circular orbits of e.g. a satellite around a black hole.

Problem 3 (Ricci-scalar don't care) [10 points]

Using the four entries $R_{\mu\mu}$ of the diagonal Ricci-tensor, calculate

$$R = g^{\mu\nu} R_{\mu\nu}$$

and plot the result against r in units of R_S. What happens at R_S?

We expect $R=0$, since we explicitly argued in exercise 1.a) that the Ricci-tensor $R_{\mu\nu}$ is zero in a vacuum system. Still we compute

$$R = g^{00}R_{00} + g^{11}R_{11} + g^{22}R_{22} + g^{33}R_{33}$$

here you have for vacuum $R_{\mu\nu} = 0$

$$= \frac{1}{B}\left[-\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A}+\frac{B'}{B}\right) - \frac{B'}{Ar}\right] - \frac{1}{A}\left[\frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A}+\frac{B'}{B}\right) - \frac{A'}{Ar}\right]$$

$$- \frac{1}{r^2}\left[\frac{1}{A} - 1 - \frac{r}{2A}\left(\frac{A'}{A}-\frac{B'}{B}\right)\right] - \frac{1}{r^2\sin^2\theta}\left[\frac{1}{A} - 1 - \frac{r}{2A}\left(\frac{A'}{A}-\frac{B'}{B}\right)\right]\sin^2\theta$$

$$= -\frac{B''}{2AB} + \frac{B'}{4AB}\left(\frac{A'}{A}+\frac{B'}{B}\right) - \frac{B'}{ABr} - \frac{B''}{2AB} + \frac{B'}{4AB}\left(\frac{A'}{A}+\frac{B'}{B}\right) + \frac{A'}{A^2 r}$$

$$- \frac{2}{Ar^2} + \frac{2}{r^2} + \frac{1}{Ar}\underbrace{\left(\frac{A'}{A}-\frac{B'}{B}\right)}_{\frac{2}{?}\left(\frac{1}{A}\right)'}$$

$$= -\frac{B''}{2AB} + \frac{B'}{2AB}\underbrace{\left(\frac{A'}{A}+\frac{B'}{B}\right)}_{\ln(A)' + \ln(B)' = \ln(A)' + \ln(A^{-1})' = \ln(A)' - \ln(A)' = 0} - \frac{2B'}{ABr} + \frac{2A'}{A^2 r} + \frac{2}{r^2}\left(1 - \frac{1}{A}\right)$$

$R = 0$!

We now use $B(r) = A^{-1}(r) = 1 - \frac{2a}{r}$ to simplify

$$R = -\left(1-\frac{2a}{r}\right)'' - \frac{4}{r}\left(1-\frac{2a}{r}\right)' + \frac{2}{r}\left(1-\frac{2a}{r}\right)' + \frac{2}{r^2}\frac{2a}{r}$$

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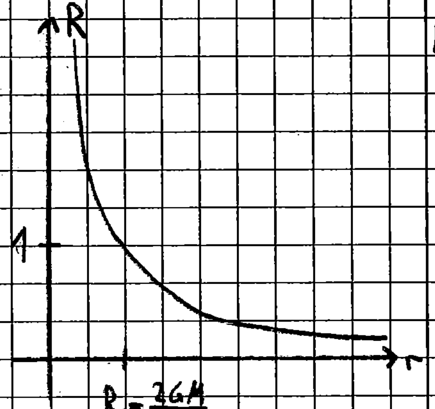
$$= \frac{4a}{r^3} + \frac{4a}{r^3} = \frac{8a}{r^3} = \frac{8GM}{c^2 r^3}$$



No unusual behavior at $r = R_S$.

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Problem 4 (Extra: Black hole detection) [5 points]

Name two ways of detecting a black hole! Since we can't see it directly, how would you go about looking for one in our cosmos?

Black holes do not emit light directly but influence their surroundings. Detection should be possible via:

- gravitional effects on nearby celestial objects where no apparent source can be found ✓!
- gravitational lensing of light traversing close to black hole ✓!
- emission of gamma rays from accretion disk around black hole ✓!
- gravitational waves discuss! ✓