String Theory - Exam Sheet

Janosh Riebesell

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1 Basics of string theory

- central axiom: fundamental objects in Nature not pointlike, but 1-dimensional (combined with standard kinematics of general covariance and usual procedure of quantization)
  - gen. cov.: inv. of form of physical laws under arb. diff. coordinate trafos.; essential idea: coordinates don't exist in nature, are only artifices in our description, hence should play no phys. role
- two sectors arise from elementary fact string can be open/closed: open = Yang-Mills, closed = gravity; since open strings can close up and vice versa, both are automatically dynamically related

2 The classical bosonic string

- $S_{\text{NG}} = -T \int_\Sigma \mathrm{d}A$ defines **Nambu-Goto action** of classical bosonic string, where T string tension, $\mathrm{d}A = \sqrt{-\det(\boldsymbol{G})}\,\mathrm{d}\tau\mathrm{d}\sigma$ area element of worldsheet (WS) Σ with coordinates $\boldsymbol{\xi} = (\tau, \sigma)$ and induced metric (or pullback of ambient space metric $\eta_{\mu\nu}$ onto Σ) $G_{ab} = \frac{\partial X^\mu}{\partial \xi^a}\frac{\partial X_\mu}{\partial \xi^b}$
- to eliminate square root in S_{NG}, introduce WS metric $h^{ab}(\tau,\sigma)$ as auxiliary field in on-shell-S_{NG}-equivalent **Polyakov action** $S_{\text{P}} = -\frac{T}{2}\int_\Sigma \mathrm{d}\tau\mathrm{d}\sigma \sqrt{-h}\, h^{ab} G_{ab}$, with $h = \det(\boldsymbol{h})$; X^μ still spacetime (ST) vector but scalar on WS, hence S_{P} simply action of d scalars
  - **symmetries**: **1.** d-dim. ST **Poincaré invariance** $X^\mu \to \Lambda^\mu{}_\nu X^\nu + V^\mu$, with $\Lambda^\mu{}_\nu \in SO(1, d{-}1)$ **2.** local WS **diffeomorphism inv.** under $\xi^a \to \xi^a - \epsilon^a(\xi)$ under which the WS scalar $\delta X^\mu = \epsilon^a \partial_a X^\mu$, the metric $\delta h_{ab} = \nabla_a \epsilon_b + \nabla_b \epsilon_a$, scalar density of weight 1 $\delta\sqrt{-h} = \partial_a(\epsilon_a\sqrt{-h})$ **3.** local **Weyl inv.** $h_{ab} \to e^{2\omega(\xi)} h_{ab}$, special symmetry only for 2-dim. WS, important for cons. quantization
  - could add 2 terms to S_{P}: **1.** cosmol. constant term $S_\Lambda = \Lambda \int_\Sigma \mathrm{d}^2\xi \sqrt{-h}$, would spoil conf. inv. **2.** Einstein-Hilbert term $S_{\text{EH}} = \frac{\lambda_{\text{EH}}}{4\pi}\int_\Sigma \mathrm{d}^2\xi \sqrt{-h}\mathcal{R}$, is a total derivative $\Rightarrow$ no dynamics; also $S_{\text{EH}} \propto \chi$ Euler char.
- **e.-m. tensor** $T_{ab} \equiv \frac{4\pi}{\sqrt{-h}}\frac{\delta S_{\text{P}}}{\delta h^{ab}} = -\frac{1}{\alpha'}(G_{ab} - \frac{1}{2}h_{ab}G^c{}_c)$, traceless $T^a{}_a = 0$ as consequence of Weyl inv., conserved current $\nabla^a T_{ab} = 0$ w.r.t. local WS diffeo. for on-shell X^μ
- **gauge fixing**: h_{ab} symmetric has $\frac{d}{2}(d+1)$ d.o.f., diffeo. + Weyl has $(d+1)$, hence for $d = 2$ where Weyl trafo. $\omega(\xi)$ s.t. $\sqrt{-h}\mathcal{R} \to \sqrt{-h}(\mathcal{R} - 2\Delta\omega) = 0$ implies $R^a{}_{bcd} = 0$, we can (locally) gauge away all metric d.o.f., then diffeo. trafo. to obtain flat WS $h_{ab} = \eta_{ab}$
  - note: leaves large **residual gauge symmetry** generated by conformal Killing vectors $\boldsymbol{\epsilon}$ satisfying $(\sum \cdot\, \boldsymbol{\epsilon})_{ab} = \nabla_a\epsilon_b + \nabla_b\epsilon_a + \nabla^c\epsilon_c h_{ab} = 0$ whose effect on metric can be undone by Weyl trafo.
- **lightcone coordinates**: $\xi^\pm = \tau \pm \sigma$; metric $h_{\pm\pm} = 0$, $h_{\pm\mp} = -\frac{1}{2}$, $h^{\pm\mp} = -2$; line element $\mathrm{d}s^2 = h_{ab}\xi^a\xi^b = -\mathrm{d}\tau^2 + \mathrm{d}\sigma^2 = -\mathrm{d}\xi^+\mathrm{d}\xi^-$
  - e.m.-tensor: $T_{\pm\pm} = -\frac{1}{\alpha'}\partial_\pm \boldsymbol{X}\cdot\partial_\pm\boldsymbol{X}$, tracelessness $T_{\pm\mp} = 0$, conservation $\partial_\mp T_{\pm\pm} = 0 \Rightarrow T_{\pm\pm}(\xi^\pm)$; crucial: in flat gauge, h_{ab}-e.o.m. $T_{ab} = 0$ still has to be enforced as constraint $T_{\pm\pm} = 0$
- **mode expansion**: varying flat gauge $S_{\text{P}} = \frac{T}{2}\int_\Sigma \mathrm{d}\tau\mathrm{d}\sigma[(\partial_\tau\boldsymbol{X})^2 - (\partial_\sigma\boldsymbol{X})^2] = T\int_\Sigma \mathrm{d}^2\xi\, \partial_+\boldsymbol{X}\cdot\partial_-\boldsymbol{X}$ yields free wave equation $(\partial_\tau^2 - \partial_\sigma^2)X^\mu = 0 = \partial_+\partial_- X^\mu$ as string e.o.m. provided b.t. vanish: cl. string ✓, open string requires Neumann $(\partial_\sigma X^\mu = 0)$ and/or Dirichlet $(\delta X^\mu = 0 = \partial_\tau X^\mu)$ b.c. at $\sigma = 0, l$; each has diff. exp., e.g. open NN string: $X^\mu = x^\mu + \frac{p^\mu\tau}{T\,l} + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}e^{-i\frac{\pi}{l}n\tau}\cos(\frac{n\pi\sigma}{l})$
  - modes fulfill comm. rel. $[\alpha_m^\mu, \alpha_n^\nu] = m\,\eta^{\mu\nu}\delta_{m,-n}$; $[x^\mu, p^\nu] = i\eta^{\mu\nu}$
  - insert resulting $\partial_\pm X^\mu$ into e.m.-tensor to get its mode expansion $T_{\pm\pm} = 4\alpha'\sum_{m\in\mathbb{Z}} L_m^\pm e^{-i\frac{2\pi}{l}m\xi^\pm}$ i.t.o. the Virasoro generators L_m; $T_{ab} = 0$ implies the **Virasoro constraints** $L_m^\pm = 0\ \forall\, m \in \mathbb{Z}$
- **Dp-brane** is $(p+1)$-dim. hypersurface on which open strings can end, fixing them in dims. normal to it; mom. exchange with string implies brane is a dynamical (albeit non-perturbative) object itself
- **Hamiltonian**: $H_{\text{op}} = \frac{\pi}{l}(\frac{1}{2}\boldsymbol{\alpha}_0^2 + \frac{1}{2}\sum_{n\neq 0}\boldsymbol{\alpha}_{-n}\cdot\boldsymbol{\alpha}_n) = \frac{\pi}{l}L_0$, must vanish since $T_{ab} = 0$ which implies **mass shell cond.**, e.g. for open string $M^2 = -\ ^2 \stackrel{!}{=} \frac{1}{\alpha'}\sum_{n=1}^\infty \boldsymbol{\alpha}_{-n}\cdot\boldsymbol{\alpha}_n$, for closed string $H_{\text{cl}} = \frac{2\pi}{l}(L_m^+ + L_m^-) \propto \partial_+ + \partial_- \propto \partial_\tau = 0$ implem. time reparametr. inv.

3 Bosonic string quantization

- 3 different ways to quantize: **1. old covariant** (OCQ): Viras. constr. implemented at quantum level; manifestly Lorentz covariant, but unitary only in critical number d_{crit} of ST dims. **2. lightcone** (LCQ): Viras. constr. implemented classically, manifestly unitary, but Lorentz covariant only in $d = d_{\text{crit}}$ **3. path-integral** (PIQ): uses Faddeev-Popov (FP) gauge fixing procedure, criticality equivalent to closure of BRST algebra, only closed in $d = d_{\text{crit}}$
- **normal ordering** $\mathcal{N}(\alpha_m^\mu\,\alpha_n^\nu) = \alpha_m^\mu\,\alpha_n^\nu$ for $m \leq n \wedge \alpha_n^\nu\,\alpha_m^\mu$ else introduces ambiguity in $L_0 \to L_0 - a$ only, captured in norm. ord. const. a interpreted as Casimir energy, fixed by consistency cond.
- **Virasoro algebra** $[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m,-n}$ is central extension by $\mathbb{C}$ of classical Witt algebra, **central charge** $c = \eta^\mu{}_\mu = d$ given by number of scalar fields X^μ; $c \neq 0$ indicates quantum anomaly of WS conformal symmetry
- **phys. state cond.** $(L_m - a\delta_{m,0})|\phi\rangle = 0\ \forall\, m \geq 0 \wedge |\phi\rangle \in \mathcal{H}_{\text{phys}}$
- **tower of states**: $M_{\text{op}}^2|\phi\rangle = \frac{1}{\alpha'}(N-a) + T^2\Delta\boldsymbol{x}^2|\phi\rangle$ with number op. N counting excitations by creators $\boldsymbol{\alpha}_n$, $n \leq 0$; $M_{\text{cl}}^2|\phi\rangle = \frac{2}{\alpha'}(N^+ + N^- - a)|\phi\rangle$ governed by level matching condition $(N^+ - N^-)|\phi\rangle = 0$
- **criticality**: string spectrum analysis reveals unitarity (OCQ)/non-anomalous Lorentz algebra (LCQ) requires $a = 1$, $d = 26$
- bosonic vacuum $|0, \ \rangle$ is **tachyonic** $M^2 = -\frac{1}{\alpha'}$; known from QFT as not inconsistent, merely signal instability of (naive) vacuum
- for general b.c.s, Casimir energy increases by $\frac{1}{24}$ per NN/DD dim. and decr. by $-\frac{1}{48}$ per ND/DN dim., i.e. $a_{\text{tot}} = \frac{d-2}{24} - \frac{n_{\text{ND}} + n_{\text{DN}}}{16}$
- open spectrum with D-branes: first-level excitations parallel to brane form **massless vector** = gauge field $\Rightarrow$ single brane hosts $U(1)$ gauge theory; normal exc. = n_{DD} massless scalars (Goldstone bosons assoc. with spontan. breaking of 26-dim. Poincaré inv.)
- N coincident branes carry $U(N)$ gauge theory; in orientifolded theories also $SO(N)$ and symplectic $Sp(2N)$ gauge groups possible
- first-level closed string polarization tensor decomposes into 3 irred. repr. of little group $SO(24)$: $\xi_{ij} = g_{ij} + B_{ij} + \phi\,\delta_{ij}$, with g_{ij} massless, transversely polarized spin 2 particle (**graviton**), B_{ij} antisymmetric (**Kalb-Ramond**) tensor field, ϕ scalar field (**dilaton**)
- **PIQ**: partition function $Z = \int \mathcal{D}X\, \det(\sum)e^{iS_{\text{P}}[X,\hat{h}]}$ with FP determinant $\det(\sum)$, arbitrary reference metric $\hat{h}$
  - can be written $Z = \int \mathcal{D}X\mathcal{D}b\mathcal{D}c\, e^{i(S_{\text{P}} + S_g)}$, by introducing FP ghost $c^a(\xi)$, antighost $b_{ab}(\xi)$ (anti-commuting, fermionic fields with integer spin, negative norm states), governed by ghost action $S_g = \frac{-i}{2\pi}\int_\Sigma \mathrm{d}^2\xi\sqrt{-\hat{h}}\hat{h}^{ab}c^d\nabla_a b_{bd} \stackrel{\text{lcg}}{=} \frac{i}{\pi}\int_\Sigma \mathrm{d}^2\xi(c^+\partial_- b_{++} + c^-\partial_+ b_{--})$ and e.o.m.s $\nabla^a b_{ab} = 0 = \partial_\mp b_{\pm\pm}$ & $\sum\cdot\,\boldsymbol{c} = 0 = \partial_\mp c^\pm \Rightarrow c^a$ in 1-to-1 corresp. with conf. Killing vecs.
- **ghost Virasoro alg.** $[L_m^g, L_n^g] = (m-n)L_{m+n}^g + \frac{m}{6}(1 - 13m^2)\delta_{m,-n}$ where $L_m^g = \sum_{n\in\mathbb{Z}}(m-n)\mathcal{N}(b_{m+n}c_{-n})$ i.t.o. anti-comm. ghost modes b_n, c_n with $\{c_m, b_n\} = \delta_{m,-n}$, $\{c_m, c_n\} = \{b_m, b_n\} = 0$
  - yields combined Virasoro alg. $[L_m^{\text{tot}}, L_n^{\text{tot}}] = (m-n)L_{m+n}^{\text{tot}} + m[\frac{c^{\text{tot}}}{12}(m^2-1) + 2(a-1)]\delta_{m,-n}$ with central charge $c^{\text{tot}} = c^X + c^g$ where $c^g = -26$, $c^X = d$ in $\mathbb{R}^{1,d-1}$, hence Weyl anomaly in PI absent iff $d = 26$, $a = 1$ (this really fixes c^X, only indirectly d)
- **BRST symmetry** generated by necessarily nilpotent cons. charge Q_B, $Q_B^2 = 0$ holds if full Viras. alg. non-anomalous ($d = 26$, $a = 1$), i.e. BRST consistency requires absence of total Weyl anomaly
  - $Q_B|\phi\rangle = 0\ \forall|\phi\rangle \in \mathcal{H}_{\text{phys}}$ is necessary phys. state cond.; pos. norm Hilbert space $\mathcal{H}_{\text{phys}} = \frac{\mathcal{H}_{\text{clos}}}{\mathcal{H}_{\text{exac}}} = \frac{\ker(Q_B)}{\text{Im}(Q_B)} \equiv Q_B$ cohomology

4 Conformal field theory

- examples: **1.** string WS is 2-dim. CFT **2.** at fixed points of RG eqs. in QFT, theory becomes scale inv. **3.** at crit. points in CMP and SP where correlation length diverges **4.** AdS/CFT corresp. relates gravity on AdS space to CFT on its boundary
- **conformal trafo.** = diffeom. that changes metric $g_{\mu\nu}(x) \to \partial_{\mu'}x^\alpha\,\partial_{\nu'}x^\beta\, g_{\alpha\beta} \stackrel{!}{=} e^{\omega(x)}g_{\mu\nu}(x)$ only by a factor, i.e. infinitesimally $\partial_\mu\epsilon_\nu + \partial_\nu\epsilon_\mu = \omega(x)g_{\mu\nu}$ if we set $x'^\mu = x^\mu - \epsilon^\mu(x)$ and $e^{\omega(x)} = 1 + \omega(x)$
  - conf. trafos. include translations, Lorentz trafos., dilations, special conf. trafos. (= inversion, translation, another inversion)
  - $\omega(x)$ satisfies constraints some of which are vacuous in $d = 2 \Rightarrow$ makes group of infinites. conf. trafos. less restrictive, its volume infinite; this allows to solve some theories exactly/completely
- group of finite conf. diffeos. $z \to \frac{az+b}{cz+d}$ on $\mathbb{S}^2$ is **Möbius group** $PSL(2,\mathbb{C}) = SL(2,\mathbb{C})/\mathbb{Z}_2$ (since $(a,b,c,d) \stackrel{\mathbb{Z}_2}{\to} (-a,-b,-c,-d)$ same trafo.); generated by l_{-1}, l_0, l_1, with $l_n = -z^{n+1}\partial_z$ (which fulfill

Witt alg. = Viras. alg. but classical, i.e. no c-term)

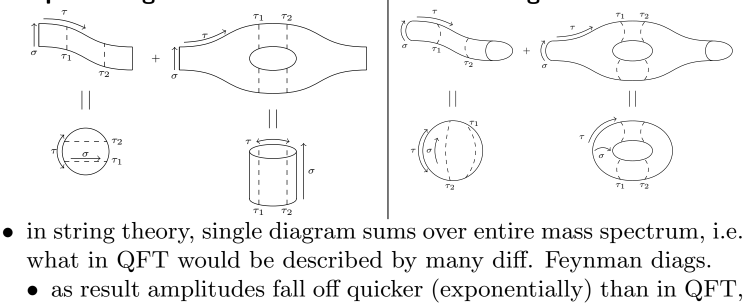
- import. prop. of $PSL(2,\mathbb{C})$: maps any 3 distinct points to any other 3, crucial since used to remove gauge redundancy by fixing positions of asymptotic in- and out-states in scattering ampl.

- **primary fields** transform as tensors $\phi(z,\bar z) \to \phi'(z',\bar z') = (\partial_z f)^{-h} (\partial_{\bar z}\bar f)^{-\bar h}\phi(z,\bar z)$ under conf. trafo. $z \to z' = f(z)$, where $(h,\bar h)$ = conf. weights ($h+\bar h$ = mass dim., $h-\bar h$ = spin)
  - infinites.: $f(z) = z + \epsilon(z) \Rightarrow \delta_{\epsilon,\bar\epsilon}\,\phi = -(h\partial_z + \epsilon\partial_z + \bar h\partial_{\bar z} + \bar\epsilon\partial_{\bar z})\phi$
  - together with (successive) operator product exps. (OPE), primaries can be used to express all higher n-point fcts. i.t.o. lower correlators; this is idea behind defining CFT i.t.o. finite amount of data, namely conf. anomaly c, spectrum of primaries ϕ_j, their weights h_j and OPE coeffs. $C_{ij}{}^k$
  - **quasi-primary field**: like primary, but only for $f \in PSL(2,\mathbb{C})$
  - applied to strings: X^μ not (quasi)primary, but ∂X^μ, $\mathcal{N}(e^{ikX})$ are
- radially ord. **OPE**: by **Wick's thm.**, $\mathcal{R}(\prod_i \phi_i) = \mathcal{N}\{\prod_i \phi_i + \sum_{j\neq k}\langle\phi_j\phi_k\rangle\prod_{i\neq j,k}\phi_i + \sum_{j\neq k}^{l\neq m}\langle\phi_j\phi_k\rangle\langle\phi_l\phi_m\rangle\prod_{i\neq j,k}^{i\neq l,m}\phi_i + \dots\}$
- **conf. Ward-Takahashi id.**: $\delta_{\epsilon,\bar\epsilon}\mathcal{O}(z,\bar z) = -\int_{C_z}\big[\frac{\mathrm{d}w}{2\pi i}\epsilon(w)T(w) + \frac{\mathrm{d}\bar w}{2\pi i}\bar\epsilon(\bar w)\bar T(\bar w)\big]\mathcal{O}(z,\bar z) \Rightarrow$ info about conf. trafos. encoded in residua of OPE with e.m.-tensor (integrand radially ordered)
  - yields OPE of $T(z)$ with primary $\phi(w)$ of weight h: $\mathcal{R}[T(z)\phi(w)] = \frac{h\,\phi(w)}{(z-w)^2} + \frac{\partial_w\phi(w)}{z-w} + \text{reg. terms}$
  - e.m.-tensor OPE follows from $[L_m, L_n]$ as $T(z)T(w) = \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial_w T(w)}{z-w} \Rightarrow T(z)$ primary of $h=2$ if $c=0$
- **operator-state correspondence**: isomorphism in 2-dim. CFT that relates primary fields to highest weight states, e.g. $|\phi\rangle = \phi(0)|0\rangle = \phi_{-h}|0\rangle$ for h-weight primary with expansion $\phi(z) = \sum_{n\in\mathbb{Z}}\phi_n z^{-n-h}$, by residue thm. $\phi_n = \int_{C_0}\frac{\mathrm{d}z}{2\pi i}\phi(z)z^{n+h-1}$
- requiring BRST-inv. for X-CFT gives phys. state cond. $L_m|\phi\rangle = 0\ \forall m > 0$ and $(L_0 - 1)|\phi\rangle = 0 \Rightarrow$ phys. states are in 1-1 corresp. with primaries of weight $h=1$; leads to concept of **vertex operator** $\equiv$ primary field of $h=1$, e.g. $\mathcal{N}(e^{i\boldsymbol{k}\cdot\boldsymbol{X}})$ with $h = \frac{\alpha'}{4}\boldsymbol{k}^2 \overset{!}{=} 1 \Rightarrow M^2 = -\frac{4}{\alpha'}$ or $\mathcal{N}[\partial X^\mu(z)e^{i\boldsymbol{k}\cdot\boldsymbol{X}(z)}]$ with $h_{V_1} = 1 + \frac{\alpha'}{4}\boldsymbol{k}^2 \overset{!}{=} 1 \Rightarrow \boldsymbol{k}^2 = 0$ inserted at $z=0$, creates first exc. level phys. state from $PSL(2,\mathbb{C})$-inv. vacuum
- **Verma module** V_{h_j} is span of all states of form $|\phi_j^{k_1\dots k_m}\rangle = \prod_i^m L_{-k_i}|\phi_j\rangle$ with ascending k_i and conf. weight $h_V = h_j + \sum_i^m k_i$
- **CFT unitarity**: holds if conformal anomaly $c > 0$ and spectrum of primaries ϕ_j fulfills $h_j \geq 0\ \forall j$ and $h_\phi = 0 \Leftrightarrow \phi = \mathbb{1}$, i.e. only $PSL(2,\mathbb{C})$-inv. vacuum may have $h=0$

5 String interactions

- no localized vertices, interac. captured by **global WS topology** $\Rightarrow$ no need to add arbitrary terms to WS action, S_P remains free
  - thus correlators of diff. fields (bosons, fermions, ghosts) decouple (unlike e.g. Yang-Mills with ghost-gauge interact.), *very* useful
- goal in string perturbation: **S-matrix** of scattering process, at each order latter may (by conformal symmetry) be described by compact WS with vertex op. insertions instead of asymptotic in-/out-states
  - S-matrix sums up all WS topols.; imp. thm. 'every compact, connected, oriented 2-dim. manifold topologically equiv. to sphere with (g,b) handles, boundaries' $\Rightarrow$ WSs classified by **Euler char.** $\chi = 2 - 2g - b$, a topol. inv. under continuous deformations of WS metric, given by **Riemann-Roch thm.** $\chi = \int_\Sigma \frac{\mathrm{d}^2\xi}{4\pi}\sqrt{-h}\mathcal{R} + \int_{\partial\Sigma}\frac{\mathrm{d}s}{2\pi}k$, Ricci sc. $\mathcal{R}$, geodesic curvat. k
  - $S_{j_i}(k_i) = \sum_{\substack{\text{comp}\\ \text{topos}}} \frac{\int\mathcal{D}X\int\mathcal{D}h}{\mathrm{Vol}_{\mathrm{Diff}\times\mathrm{Weyl}}} e^{-S_\mathrm{P} - \lambda\chi}\prod_{i=1}^n V_{j_i}(k_i)$ heur. expr. (before gauge fixing) for n string scattering, added χ-term to action (without affecting dynamics) to keep track of topol. in PI
- e.g. tree-level and one-loop topologies: disk $\mathbb{D}^2$ $[(0,1),\ \chi=1]$, cyl. $\mathbb{C}^2$ $[(0,2),\ \chi=0]$, sphere $\mathbb{S}^2$ $[(0,0),\ \chi=2]$, torus $\mathbb{T}^2$ $[(1,0),\ \chi=0]$

Open string **Closed string**



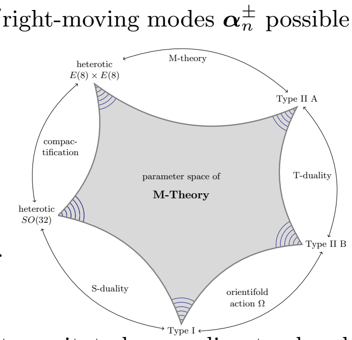
- in string theory, single diagram sums over entire mass spectrum, i.e. what in QFT would be described by many diff. Feynman diags.
  - as result amplitudes fall off quicker (exponentially) than in QFT, partially respons. for **UV finiteness** of string loop diags.
  - another reason: **modular inv.** [under action of modular group, e.g. $PSL(2,\mathbb{Z})$ on the torus] acts as intrinsic UV cutoff by excluding divergent region of moduli space from fundamental domain
  - some UV divergences arise but no issue for UV finit. due to WS duality between open/closed channel, all can be reinterpreted as IR diver. of dual diagr. e.g. cylinder: tree-level cl., one-loop open
- def. **metric moduli**: deformation of metric that cannot by undone by diffeo. or Weyl resc.; by R.-R.-thm. number $\mu = \dim(\ker P^\dagger)$ of moduli and $\kappa = \dim(\ker P)$ of conf. Killing vects. fulfill $\mu - \kappa = -3\chi$ (if $\chi > 0 \Rightarrow \mu = 0$, if $\chi < 0 \Rightarrow \kappa = 0$)
- **non-linear σ-model** describes strings propagating on curved background (generated by coherent state of its *own* massless fluctuations); consistency to first order in $\frac{\alpha'}{R_c}$ yields Einstein eqs. for background metric (R_c typical radius of target space)

6 Superstring theory

- remedies tachyon-vacuum, lack of fermionic excitats. of bosonic th.
- obtained by adding $S_\mathrm{F} = -\frac{i}{4\pi}\int_\Sigma \mathrm{d}^2\xi\,\bar\psi^\mu_A\gamma^\alpha_{AB}\partial_\alpha\psi_{B,\mu} \overset{\text{lcg}}{=} \frac{i}{2\pi}\int_\Sigma\mathrm{d}^2\xi(\boldsymbol{\psi}_+\cdot\partial_-\boldsymbol{\psi}_+ + \boldsymbol{\psi}_-\cdot\partial_+\boldsymbol{\psi}_-)$ to S_P; $\psi_\pm$ are Grassmann-valued **Majorana-Weyl spinors** (real, definite chirality) with **Dirac eq.** as e.o.m. $\gamma^\alpha\partial_\alpha\psi = 0 = \partial_\mp\psi_\pm$; mass dim. $[\psi] = \frac{1}{2}$ $([X] = -1)$
- features **supersymmetry** $\delta X^\mu = i\frac{\sqrt{\alpha'}}{\sqrt2}\bar\epsilon_A\psi^\mu_A = i\frac{\sqrt{\alpha'}}{\sqrt2}(\epsilon_+\psi^\mu_- - \epsilon_-\psi^\mu_+)$, $\delta\psi^\mu_A = \frac{\epsilon_B}{\sqrt{2\alpha'}}\gamma^\alpha_{AB}\partial_\alpha X^\mu = \pm\frac{\sqrt2}{\sqrt{\alpha'}}\epsilon_\mp\partial_\pm X^\mu$; related to Poincaré by $\{Q_A,\bar Q_B\} \cong 2\gamma^a_{AB}P_a$ (laxly SUSY^2 = translation)
- generators of **super conformal symmetry**: e.-m. tensor $T_{\pm\pm} = -\frac{1}{\alpha'}\partial_\pm\boldsymbol{X}\cdot\partial_\pm\boldsymbol{X} - \frac{i}{2}\boldsymbol{\psi}_\pm\cdot\partial_\pm\boldsymbol{\psi}_\pm$ and **supercurrent** $J_\pm = \frac{-1}{2\alpha'}\boldsymbol{\psi}_\pm\cdot\partial_\pm\boldsymbol{X}$
  - **super-Viras. constr.**: $T_{\pm\pm} \overset{!}{=} 0$, $J_\pm \overset{!}{=} 0$ imposed on e.o.m. sols.
- local diffeo. inv. + supersymmetry = local supersym. $\Rightarrow$ **supergravity** in which also metric h_{ab} has superpartner, the **gravitino**
- local e.o.m. needs boundary terms to vanish; closed string b.c.s that not mix $\psi_\pm$ and respect Poincaré sym. are $\psi_\pm(\sigma + l) = e^{2\pi i\phi_\pm}\psi_\pm(\sigma)$
  - $\phi_\pm = 0\,(\frac12)$: (anti-)per. **Ramond (Neveu-Schwarz) sec.** with (half-)integer mode exp. $\boldsymbol{\psi}_\pm(\xi^\pm) = \sqrt{\frac{2\pi}{l}}\sum_{n\in\mathbb{Z}(+\frac12)}\boldsymbol{b}^\pm_n e^{-i\frac{2\pi}{l}n\xi^\pm}$
  - R-R and NS-NS bosonic; R-NS and NS-R fermionic excitations
- **GSO projection**: CFT consistency + stability of vacuum (= no tachyon) $\Rightarrow$ **Type II A/B** as closed oriented superstring theories
  - equal number of bosons + fermions, $128 + 128$ at massless level
  - 2 spin 3/2 fields (gravitino) $\Rightarrow$ low-E-limit of Type II is SuGra
  - WS consistency + vacuum stability imply local SUSY in $d = 10$
- **Type I$_\mathrm{cl}$ th.** unstable like bosonic theory due to tachyonic vacuum, inconsistent at 1-loop level due to appearance of tadpole
- only 3 consist. superstr. ths. in $d = 10$: **Type II A/B** and **Type I** of closed + unoriented open strings with gauge group $SO(32)$

7 Compactification, T-duality, D-branes

- compactification in superstring theory is the op. $\mathbb{R}^{1,9} \to \mathbb{R}^{1,3} \times \mathcal{M}^6$ with $\mathcal{M}^6$ called **internal space**; flat scalar fields whose VEV determine geometric properties of $\mathcal{M}^6$ called **moduli fields**
- truly stringy **winding states** around compact. dimensions with mass $M^2 = \frac{\omega^2R_c^2}{\alpha'^2}$ and indep. left-/right-moving modes $\boldsymbol{\alpha}^\pm_n$ possible
- **T-duality**: $n \leftrightarrow \omega$, $R \leftrightarrow R' = \frac{\alpha'}{R}$ is exact symmetry of closed CFT that affects $_L \to {}_L \propto \boldsymbol{\alpha}^-_0$, $_R \to -{}_R \propto \boldsymbol{\alpha}^+_0$, i.e. parity on right-movers
- fig.: parameter space of string th., edges are weakly coupled, interior $d = 11$ M-theory with coupling of order 1, at low energies described by supergravity



- **D-branes**: dynamical objects that gravitate by coupling to closed strings in NS-NS sector, i.e. have mass; are charged under R-R p-form potentials
- worldvolume of D-branes not static, exhibit quantum fluctuations in normal directions described by scalar light open-string excitations
- intersecting **brane worlds**: important in string phenomenology to make contact between $d = 10$ and $\mathbb{R}^{1,3}$; stack of two branes D_A, D_B intersecting along $\mathbb{R}^{1,3}$ gives rise $U(N_A)\times U(N_B)$ Yang-Mills th. + 1 chiral fermion transf. in bifundamental $(\bar N_A, N_B)$, i.e. structure of SM $SU(3)\times SU(2)\times U(1)_Y$ for $N_A = 3$, $N_B = 2$, $N_C = 1$
- every 4-dim. eff. th. obtained by compactif. corresp. to diff. choice of vacuum; tog. all solutions called **landscape of string vacua**