Assignment 7

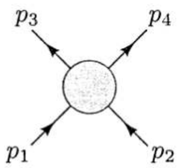
Due: Week beginning 01.06.2015.

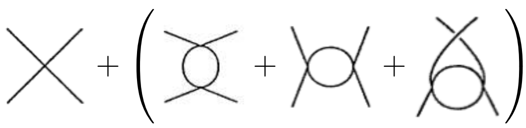
Problem 7.1 (One-loop structure of ϕ^4-theory):
Recall the Feynman rules for ϕ^4-theory including counterterms and the computation of loop diagrams via dimensional regularisation, cf. Sections 2 and 5.4.1-3 in the QFT I script from last semester. Calculate up to one-loop order, including the counterterms, the contribution for the renormalisation of:

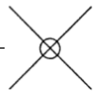
a) the propagator, i.e. $iM^2(p^2)$ of the boson propagator in momentum space

$$\frac{i}{p^2 - m^2 - M^2(p^2)}\,;$$

b) the vertex, i.e. the two-particle scattering amplitude $i\,\mathcal{M}(p_1\,p_2 \to p_3\,p_4)$

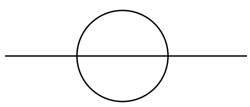
$i\,\mathcal{M}(p_1\,p_2 \to p_3\,p_4) =$ 

$=$  $+$

$+$  $+ \ldots = -i\,\lambda + (-i\,\lambda)^2(i\,V(s) + i\,V(t) + i\,V(u)) - i\,\delta_\lambda +$ higher order

up to one-loop order, where s, t and u are the Mandelstam variables.

Problem 7.2 (Field-strength renormalisation in ϕ^4-theory):
The two-loop contribution to the propagator in ϕ^4-theory involves the three diagrams shown in Figure 1. Compute the first of these diagrams in the limit of zero mass for the scalar field, using dimensional regularisation. Show that, near $d = 4$, this diagram takes the form:

 $$= -i\,p^2\,\frac{\lambda^2}{12\,(4\pi)^4}\left(-\frac{1}{\epsilon} + \log p^2 + \ldots\right)\,, \qquad (1)$$

with $\epsilon = 4 - d$. The coefficient in this equation involves a Feynman parameter integral that can be evaluated by setting $d = 4$. Verify that the second diagram in Figure 1 vanishes near $d = 4$ (and $m \to 0$ as above). Thus the first diagram should contain a pole only at $\epsilon = 0$, which can be cancelled by a field-strength renormalisation counterterm.

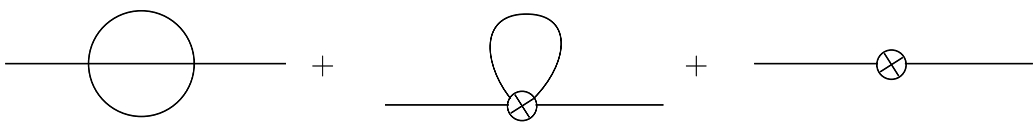


Figure 1: Two-loop diagram and counterterms for field-strength renormalisation in ϕ^4-theory.