Assignment 5

Due: Week beginning 18.05.2015.

Problem 5.1 (Diagrammetic route to the Coleman-Weinberg potential):

In Assignment Sheet 4, we looked at how to compute the Coleman-Weinberg potential, to one-loop, for a ϕ^4 theory, using path integral techniques. This week we compare this result to that computed from only using the Feynman diagrams. Recalling that

$$S[\phi] = \int d^4x \, \left(-\tfrac{1}{2}\phi(x)(\partial^2 + m^2)\phi(x) - \tfrac{\lambda}{4!}\phi^4(x)\right), \tag{1}$$

a.) Draw all 1-loop diagrams in this ϕ^4 theory (with an arbitrary number of external legs), up to λ^3.

b.) Show that

$$\Gamma[\phi] \simeq S[\phi] - i \ln \int \mathcal{D}\varphi \exp\left(-\frac{1}{2}\int dx\, dy\, \varphi(x)\, G_0^{-1}(x, y; \phi)\, \varphi(y)\right) = S[\phi] + \frac{i}{2}\text{Tr}\ln\left(G_0^{-1}(\phi)\right) \tag{2}$$

with

$$G_0^{-1}(x, y; \phi) = i\left(\partial^2 + m^2 + \frac{\lambda}{2}\phi^2(x)\right)\delta(x - y) \equiv G_0^{-1}(x, y; \phi = 0) + i\frac{\lambda}{2}\phi^2(x)\delta(x - y). \tag{3}$$

Hint: You may recall $\ln(\det A) = \text{Tr}(\ln A)$.

c.) Writing

$$\ln\left(G_0^{-1}(\phi)\right) = \ln\left(G_0^{-1}(0)\right) + \ln\left(G_0(0)G_0^{-1}(\phi)\right) \tag{4}$$

with

$$\left(G_0(0)G_0^{-1}(\phi)\right)(x, y) = \delta(x - y) + G_0(x, y; 0)\, i\frac{\lambda}{2}\phi^2(y) \tag{5}$$

expand the logarithm and identify your diagrams in part a.) with the terms in your expansion.

d.) Verify that, indeed, this reproduces the $V^{\text{eff}} = V^{\text{tree}} + V^{1-\text{loop}}$ of last week's Assignment.

Problem 5.2 (For discussion in next week's tutorial):

a.) Work through section 6.3 of Peskin and Schröder focussing on how the Wick rotation is used.

b.) Also, look through pages 289-294 of the book for the comparison to Statistical Physics.